① lec (09)

ch(4)

work & potential

outline

1. Dipole
2. ring
3. Disc
4. sphere (conductor & nonconductor)
5.

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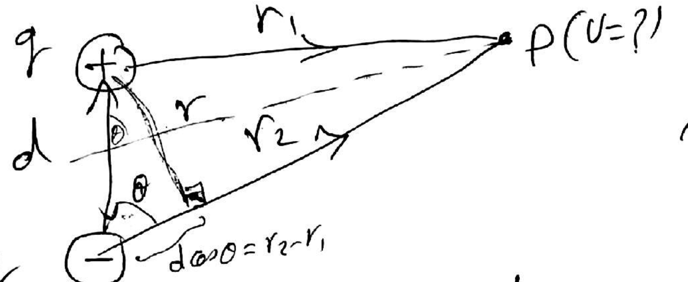
1 dipole



شحنتين عكس بعض بينهم d
معلومة ما، أحسب جهد بعيد عن الشحنتين

Find potential due to dipole at point P

$$V_+ = \frac{Q}{4\pi\varepsilon_0 r_1} \qquad V_- = \frac{-Q}{4\pi\varepsilon_0 r_2}$$

$$\therefore V_t = V_+ + V_- = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{Q}{4\pi\varepsilon_0}\,\frac{r_2 - r_1}{r_1 r_2}$$

النقطة بعيدة جداً

let $\underset{8}{r_1}\,\underset{10}{r_2} \approx \underset{(9)^2}{r^2}$ for $r \gg d$

$r_2 - r_1 \approx d\cos\theta$ $\therefore V_t = \dfrac{Q}{4\pi\varepsilon_0} \cdot \dfrac{d\cos\theta}{r^2}$

$$\boxed{V_t = QK\,\frac{d\cos\theta}{r^2}}$$ approximately

Note: $E = -\nabla V = -\left(\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi\right)$

or $= -\left(\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right)$

$= -\left(\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z\right)$

$$E = -\left(-2\frac{KQd\cos\theta}{r^3}\hat{a_r} - \frac{KQd\sin\theta}{r^3}\hat{a_\theta}\right]$$

$$= \frac{KQd}{r^3}\left[2\cos\theta\,\hat{a_r} + \sin\theta\,\hat{a_\theta}\right]$$

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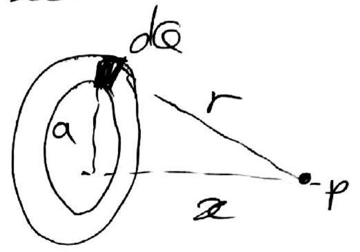
2 Ring

لإيجاد الجهد الناتج عن حلقة مشحونة Q
عند نقطة على بعد x من المركز



نأخذ عنصر من الحلقة شحنته (dQ) ونوجد الجهد x
وهذا الجهد هو dV

$\frac{KQ}{r}$ الجهد عند نقطة

فرق الجهد $KQ\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$

$$dV = \frac{dQ}{4\pi\epsilon_0 r} = K\frac{dQ}{\sqrt{a^2+x^2}}$$

$$V = \int K\frac{dQ}{\sqrt{a^2+x^2}} = \frac{K}{\sqrt{a^2+x^2}}\int dQ$$

طول المسافة ثابت على الحلقة
a , x ثوابت

$$\therefore\ V = \frac{KQ}{\sqrt{a^2+x^2}}$$

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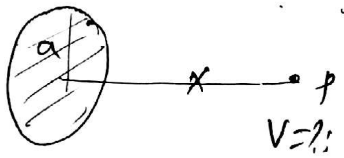
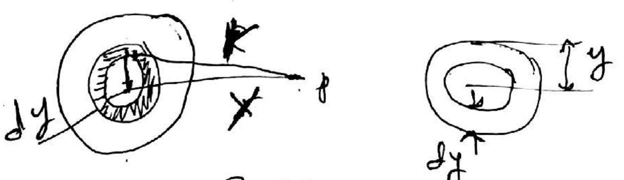
3 Charged disc $\rightarrow$ calc V

(لإيجاد ρ_s ديسك area = ρ_s)

$\rho_s \rightarrow$ surface charge density C/m^2

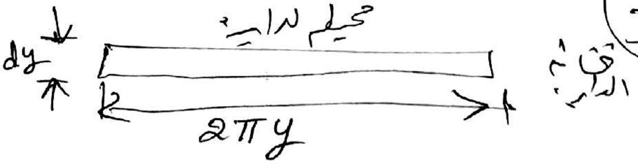


- ديسك مشحون قطره a وشحنته q
في كل متر مربع له شحنة ρ_s



- هنا نأخذ جزء من الديسك (دائرة صغيرة)

$$\rho_s = \frac{dq}{area}$$

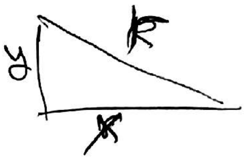


dq : شحنة الشريحة

$$dq = area \times \rho_s$$
$$= (2\pi y)(dy)\cdot\rho_s$$

$\therefore$ Potential due to this ring (dq) at point p

$$dV = \frac{dq}{4\pi\epsilon_0 R} = K\frac{dq}{\left(\sqrt{y^2+x^2}\right)}$$



$$dV = \cancel{K}\frac{(2\pi y)(dy)\,\rho_s}{4\pi\epsilon_0\sqrt{x^2+y^2}}$$

$$dV = \frac{\rho_s}{2\epsilon_0}\,\frac{y\,dy}{\sqrt{y^2+x^2}}$$

$$V = \frac{\sigma}{2\epsilon_0}\int_0^a \frac{y}{\sqrt{y^2+x^2}}\,dy = \frac{\rho_s}{2\epsilon_0}\left(y^2+x^2\right)^{\frac{1}{2}}\Big|_0^a$$

$$V = \frac{\rho_s}{2\epsilon_0}\left[\left(a^2+x^2\right)^{\frac{1}{2}} - x\right]$$

طريقة أخرى هنبدأ من الـ ring كأنه element ونكامل عليه لحد الـ area كلها

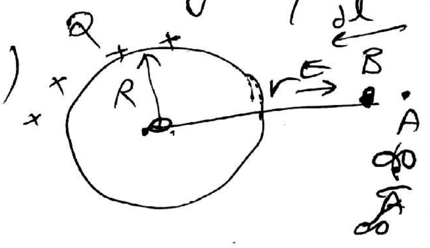
$\therefore dV = K\frac{dq}{\sqrt{x^2+y^2}}$ هنبدل a بـ y (نصف قطر الـ ring الصغير)

وتكامل كذا

4 Electric Potential due to Solid Conducting sphere

هطلع V عند نقطة B (R = نصف القطر)

(دائمًا V بتجيب كونها سكالر)



1. inside $E = 0$
2. outside $E = K\frac{Q}{r^2}$

1 outside $r > R$

لما نكون فرق الجهد بين نقطتين
نقدر نفترض البداية ∞

$$\Delta V = V_{final} - \cancel{V_i} = V_B - \cancel{V_\infty} = -\int_\infty^B E\cdot dl$$

(صفر لأن اللانهاية)

$dl = -dr$: (dl اتجاهه من ∞ لـ B) يعني dl عكس dr

E و r و dl عكس بعض
من A لـ B (من ∞ لـ B ده dl)

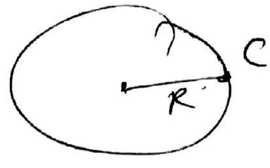
$$V_B = -\int_\infty^B \frac{KQ}{r^2}\,dl\cos(180)$$

$$= +\int_\infty^B \frac{KQ}{r^2}\,dl = KQ\int_\infty^r \frac{-dr}{r^2} = \frac{Q}{4\pi\epsilon_0}\frac{1}{r}$$

لما تفتح هتبدأ من ∞ لحد B
r ... r / B ...

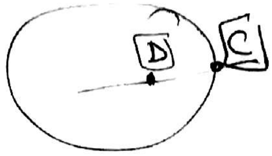
كأنه point charge

at radius of sphere



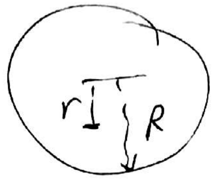
$$\boxed{V_C = \frac{kQ}{R}} \qquad \boxed{\text{at } r = R}$$

② inside $(r<R)$



E inside sphere $= 0$

$$\Delta V = -\int_C^D E \cdot dl = 0$$

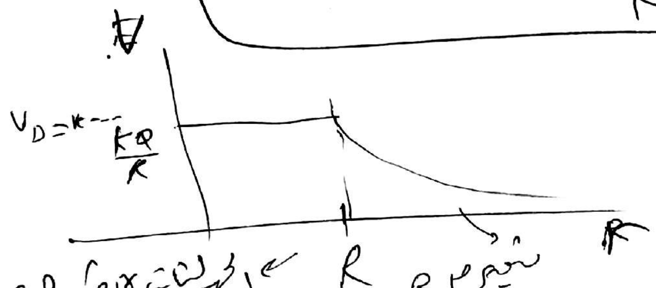


$V_D - V_C$

$$\Delta V = V_D - V_C = \frac{KQ}{R}$$

V inside sphere

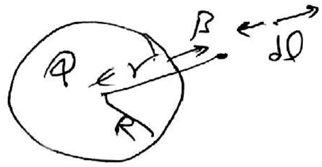
$$\boxed{V_D = V_C = \frac{KQ}{R}}$$



insulator

(5) Electric potential due to solid nonconducting sphere

ذات كثافة شحنة منتظمة داخل وخارج



Sol

$$E_{inside} = \frac{Q}{4\pi\epsilon_0 R^3} r$$

[illegible] Pot [illegible]

(1) outside

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

زي الشحنة النقطية

$$V_B - V_\infty = -\int_\infty^B \vec{E}\cdot d\vec{l} = -\int_\infty^B E\,dl\cos 180 = +\int_\infty^B E\,dl$$

$$= -\int_\infty^r E\,dr = -\int_\infty^r \frac{Q}{4\pi\epsilon_0 r^2}\,dr$$

$$= -\frac{Q}{4\pi\epsilon_0}\int_\infty^r \frac{dr}{r^2} = \boxed{+\frac{Q}{4\pi\epsilon_0 r}}$$

نفس النقطة

(2) inside

$$V_C = \frac{Q}{4\pi\epsilon_0 R}$$

سطح الكرة

$$V_D - V_C = -\int_C^D \vec{E}\cdot d\vec{l} = +\int_R^r E\,dl \;\; (\cos 180) = -\int_R^r E_{in}\,dr$$

$$V_D - V_C = -\int_R^r \left(\frac{Qr}{4\pi\epsilon_0 R^3}\right) dr = -\frac{Q}{4\pi\epsilon_0 R^3}\int_R^r r\,dr$$

$\frac{Qr}{4\pi\epsilon_0 R^3} \to E$

من قانون جاوس

$$V_D - V_C = -\frac{Q}{4\pi\epsilon_0 R^3}\left[\frac{r^2}{2}\right]_R^r = \frac{Q}{8\pi\epsilon_0 R^3}\left[R^2 - r^2\right]$$

$V_C \to \frac{Q}{4\pi\epsilon_0 R}$

$$V_D = \frac{Q}{4\pi\epsilon_0 R} + \frac{Q}{8\pi\epsilon_0 R^3}\left(R^2 - r^2\right)$$

$$\boxed{V_D = \frac{Q}{8\pi\epsilon_0 R}\left[3 - \frac{r^2}{R^2}\right]}$$

how to calc. E from ΔV

$\because \Delta V = -\int E\, dl \quad \therefore \boxed{E = -\nabla V}$

scaler (V)

Del / nabla / gradiant (operator) (∇)

$$\nabla = \frac{\partial \hat{ax}}{\partial x} + \frac{\partial \hat{ay}}{\partial y} + \frac{\partial \hat{az}}{\partial z}$$

$$-\nabla V = -\left[\frac{1}{h_1}\frac{\partial V}{\partial x}\hat{ax} + \frac{1}{h_2}\frac{\partial}{\partial y}\hat{ay} + \frac{1}{h_3}\frac{\partial}{\partial z}\hat{az}\right] \longrightarrow \text{Cartesian}$$

$$= -\left[\frac{\partial V}{\partial r}\hat{ar} + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{a\phi} + \frac{1}{\partial z}\hat{az}\right] \longrightarrow \text{Cylindrical}$$

$$= -\left[\frac{1}{1}\frac{\partial V}{\partial r}\hat{ar} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a\phi}\right] \longrightarrow \text{Spherical}$$

EX) $V = 2x^2y - 5z$, $P(-4, 3, 6)$

find Pot. at (P) & E at (P) , $|E|$, direction of E , D , ρ_v

~~$E = -\nabla V$~~ =

تعويض باحداثيات النقطة

① $V = 2(-4)^2(3) - 5(6) = 66\,V$ ✓

② $E = -\nabla V = -\left[\frac{\partial V}{\partial x}\hat{ax} + \frac{\partial V}{\partial y}\hat{ay} + \frac{\partial}{\partial z}\hat{az}\right]$

$E = -\left[4xy\,\hat{ax} + 2x^2\,\hat{ay} - 5\,\hat{az}\right]$

$= -\left[-48\,\hat{ax} + 32\,\hat{ay} - 5\,\hat{az}\right]$

$= +48\,\hat{ax} - 32\,\hat{ay} + 5\,\hat{az}$

③ $|E| = \sqrt{(48)^2 + (-32)^2 + (5)^2} = 57.9\ V/m$

④ direction of $\bar{E} = \frac{\bar{E}}{|\bar{E}|} = 0.829\,\hat{ax} - 0.55\,\hat{ay} + 0.0869\,\hat{az}$

⑤ $D = \varepsilon_0 \bar{E} = -35.4\,xy\,\hat{ax} - 17.7x^2\,\hat{ay} + 44.3\,\hat{az}\ \ pC/m^3$

⑥ $\rho_v = \nabla \cdot D = \left(\frac{\partial}{\partial x}\hat{ax} + \frac{\partial}{\partial y}\hat{ay} + \frac{\partial}{\partial z}\hat{az}\right) \cdot \left(D_x\,\hat{ax} + D_y\,\hat{ay} + D_z\,\hat{az}\right)$

$$D = -35.4xy\,\hat{a_x} - 17.7x^2\,\hat{a_y} + 44.3\,\hat{a_z}$$

$$\rho_v = \frac{\partial D_x}{\partial x} + \cancel{\frac{\partial}{\partial y} D_y}_{\to 0} + \cancel{\frac{\partial}{\partial z} D_z}_{\to 0}$$

$$\rho_v = \nabla \cdot D = -35.4y \quad Pc/m^3$$

Ex(2)

$$V = 3\ln(x^2 + 2y^2 + 3z^2)$$

find E

$$E = -\nabla V$$

$$= -\left(\left(\frac{6x}{x^2+2y^2+3z^2}\right)\hat{a_x} + \left(\frac{12y}{x^2+2y^2+3z^2}\right)\hat{a_y} + \frac{18z}{x^2+2y^2+3z^2}\right)\hat{a_z}$$

$\boxed{\text{curl}}$

$$\nabla \times \bar{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \hat{a_{u_1}} & h_2 \hat{a_{u_2}} & h_3 \hat{a_{u_3}} \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 A_{u_1} & h_2 A_{u_2} & h_3 A_{u_3} \end{vmatrix}$$

u_1	u_2	u_3	h_1	h_2	h_3
x	y	z	1	1	1
r	θ	ϕ	1	r	1
r	ϕ	z	1	r	$r\sin\theta$

div

$$\nabla \cdot \bar{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}(h_2 h_3 A_{u_1}) + \frac{\partial}{\partial u_2}(h_1 h_3 A_{u_2}) + \frac{\partial}{\partial u_3}(h_1 h_2 A_{u_3})\right]$$

grad

$$\nabla \bar{A} = \frac{1}{h_1}\frac{\partial A}{\partial u_1}\hat{a_{u_1}} + \frac{1}{h_2}\frac{\partial A}{\partial u_2}\hat{a_{u_2}} + \frac{1}{h_3}\frac{\partial A}{\partial u_3}\hat{a_{u_3}}$$

1

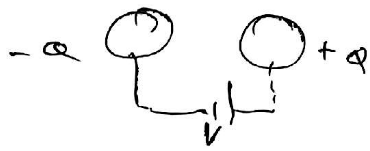
Lec (10)

Capacitors ← التحريض العام

لو عندي 2 metals و وصلت بينهم بطارية ∴ كل معدن هلاقي عليه شحنة واحد Q والتاني -Q

∴ تيار اسم والشكل ده نطلق عليه مكثف عليه شحنة

(Q الموجب والسالب عليه Q والتاني عليه -Q)



∴ $C_{capacitance} = \frac{Q}{V}$ سعة المكثف

سعة المكثف ∴ الفرق الجهد بين طرفي المكثف = 1 فولت ∴ يشيل شحنة مقدارها الواحد

وتكون وحدة farad وفي تكون micro / pico / nano

∴ any Two conductors embedded in a homogenous dielectric

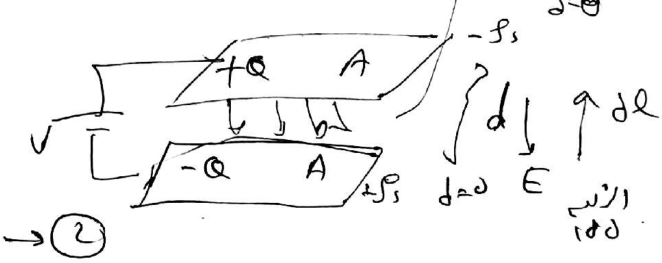
أي موصلين مغموسين وسط عازل

ومن فوائد المكثف في دوائر الـ rectifiers ل دوائر الـ radio (Tunning)

Capacitor types
- (a) parallel-plate capacitors
- (b) cylindrical "
- (c) spherical "

اعتبر اول الم العازل هواء ($\varepsilon_r = 1$)

[a]- Parallel plate capacitors



$C = \frac{Q}{V}$ → (1)

$Q = \int \rho_s \cdot dA = \rho_s * area$ → (2)

∴ $E = \frac{V}{d}$ $\frac{\text{فرق الجهد (potential)}}{\text{distance}}$ → (3) ∵ electric field uniform

$V = -\int^{+} E \cdot dl$
$= -E\, d \cos 180 = +Ed$

∴ $C = \frac{Q}{V} = \frac{\rho_s * area}{E \cdot d} = \frac{\rho_s A}{E \cdot d}$

∵ $E = \frac{\rho_s}{2\varepsilon} \times 2 = \frac{\rho_s}{\varepsilon_0}$

نتيجة مجموع الحقلين

$E_2 + E_1 = E$
$= \frac{\rho_s}{2\varepsilon_0} + \frac{\rho_s}{2\varepsilon_0}$

∴ $\boxed{C = \frac{\rho_s A}{\frac{\rho_s}{\varepsilon_0} d} = \frac{\varepsilon_0 A}{d}}$

$\varepsilon = \varepsilon_0 \varepsilon_r$

$Q_{encl} = \oint \varepsilon E \cdot ds$

$V_0 = -\int^{+} E \cdot dl$ من السالب للموجب

if $Q \uparrow \rightarrow D \uparrow \rightarrow E \uparrow \rightarrow V_0 \uparrow \rightarrow C$ const

EX(1) Calc. The Capacitance of Parallel plate Capacitor having a mica dielectric $\epsilon_r = 6$, a plate area of $10\ in^2$ & a separation of $0.01\ in$

Sol

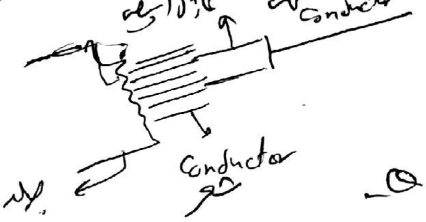
$$\epsilon = \epsilon_r \epsilon_o = 6 \times 8.85 \times 10^{-12}$$

$$A = 10 \times \left(\frac{2.5}{100}\right)^2 = 6.45 \times 10^{-3}\ m^2$$

$$d = 0.01 \times \left(\frac{2.5}{100}\right) = 2.54 \times 10^{-4}\ m$$

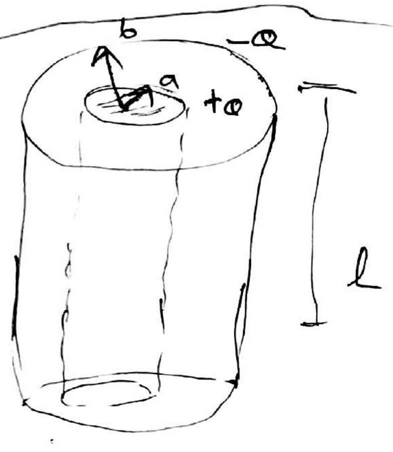
$$C = \frac{\epsilon A}{d} = \frac{6 \times 8.85 \times 10^{-12} \times 6.45 \times 10^{-3}}{2.54 \times 10^{-4}} = 1.349\ nF$$

(b) Cylindrical Capacitor



زي كابل الدش

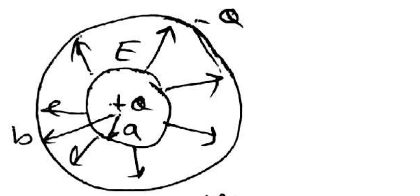
اتجاه E من +Q لـ -Q



$$C = \frac{Q}{\Delta V} \rightarrow (\Delta V)\ \text{فرق الجهد بين الموصلين}$$

$$V = -\int E \cdot dl$$

مطلوب حساب E بين الموصلين



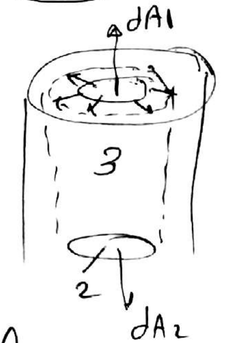
هنستخدم قانون جاوس بفرض سطح جاوسي

$$\oint E \cdot dA = \frac{Q_{en}}{\epsilon_o}$$

٣ سطوح للاسطوانة

1، 2 الـ dA عمودين على E

افقي موازي ∴ صفر 1، 2

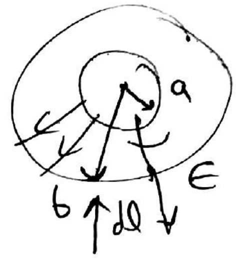


$$\Rightarrow \oint E_3 \cdot dA = \frac{Q_{en}}{\epsilon_o}$$

$$E(2\pi r l) = \frac{+Q}{\epsilon_o} \quad \therefore \quad \boxed{E = \frac{Q}{2\pi \epsilon_o r L}}$$

$$E = \frac{\rho_L}{2\pi \epsilon_o r} \Rightarrow \text{line charge}$$

③



$\because V_a - V_b = -\int_b^a \frac{Q}{2\pi\epsilon_0 r L}\, dl$ → من b لـ a

$\Rightarrow V_a - V_b = \int_b^a \frac{Q}{2\pi\epsilon_0 r l}\, dl \cos 180$ → [illegible]

$= + \int_b^a \frac{Q}{2\pi r l \epsilon_0} (-dr)$

$\therefore V_a - V_b = -\frac{Q}{2\pi l \epsilon_0} (\ln a - \ln b)$

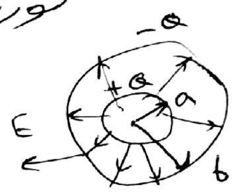
$V_a - V_b = +\frac{Q}{2\pi l} (\ln b - \ln a) = \frac{Q}{2\pi l \epsilon_0} \ln \frac{b}{a}$

$$\boxed{V_a - V_b = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{b}{a}}$$

$$\boxed{C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{2\pi\epsilon_0 l} \ln \frac{b}{a}} = \frac{2\pi\epsilon_0 l}{\ln b/a}}$$

(C) Spherical Capacitor

كورتين داخل بعض



$C = \frac{Q}{\Delta V}$

$\Delta V = -\int E \cdot dl$

هنا هنحط gauss surface ع E ∴ [illegible]

$\oint E \cdot dA = \frac{Q_{en}}{\epsilon_0} = \frac{+Q}{\epsilon_0}$

$E \cdot 4\pi r^2$

$\therefore \boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}}$

$\therefore \Delta V = -\int_b^a E \cdot dl = -\int_b^a \frac{Q}{4\pi\epsilon_0 r^2} \cdot dR$

$= -\int_b^a \frac{Q}{4\pi\epsilon_0 r^2} \cdot (+dR \cos 180) = + \int_b^a \frac{Q}{4\pi\epsilon_0 r^2} (-dr)$

$$\boxed{\Delta V = + \frac{Q}{4\pi\epsilon_0 r}\Big|_b^a} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q}{4\pi\epsilon_0} \left(\frac{b-a}{ab}\right)$$

$$\boxed{C = \frac{Q}{\Delta V} = \frac{4\pi\epsilon_0\, ab}{b-a} = \frac{1}{k}\,\frac{ab}{b-a}}$$

(4)

خلاصة الكلام

قانون عام لأي مكثف

$$C = \frac{Q}{\Delta V}$$

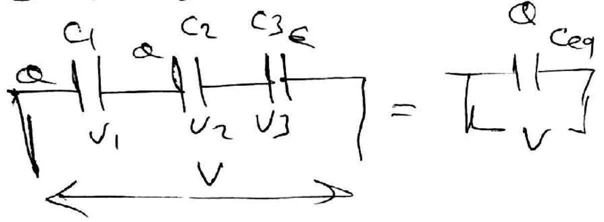
parallel plate	cylindrical	spherical
$C = \frac{\varepsilon_0 A}{d}$	$C = \frac{2\pi\varepsilon_0 l}{\ln b/a}$	$C = \frac{4\pi\varepsilon_0 ab}{b-a}$

قوانين خاصة حسب شكل المكثف هندسيا

طرق توصيل المكثفات في الدوائر

series

الشحنة واحدة = الجهد مختلف



$V = V_1 + V_2 + V_3$

الشحنة واحدة والجهد مختلف للمكثفات

$$\frac{\cancel{Q}}{C_{eq}} = \frac{\cancel{Q}}{C_1} + \frac{\cancel{Q}}{C_2} + \frac{\cancel{Q}}{C_3}$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

لو المكثفات متساوية identical

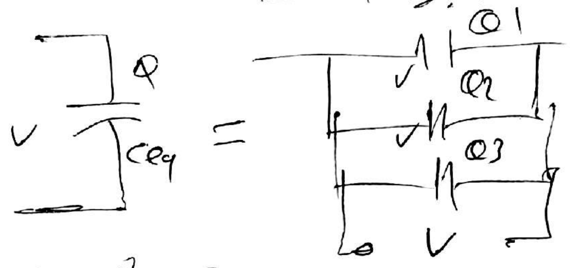
$$\frac{1}{C_{eq}} = \frac{n}{C}$$

$$\boxed{C_{eq} = \frac{C}{n}}$$

Parallel

الشحنة مختلفة = الجهد واحد

الشحنة مختلفة والجهد واحد



$Q = Q_1 + Q_2 + Q_3$

$$C_{eq}\cancel{V} = C_1\cancel{V} + C_2\cancel{V} + C_3\cancel{V}$$

$$\boxed{C_{eq} = C_1 + C_2 + C_3}$$

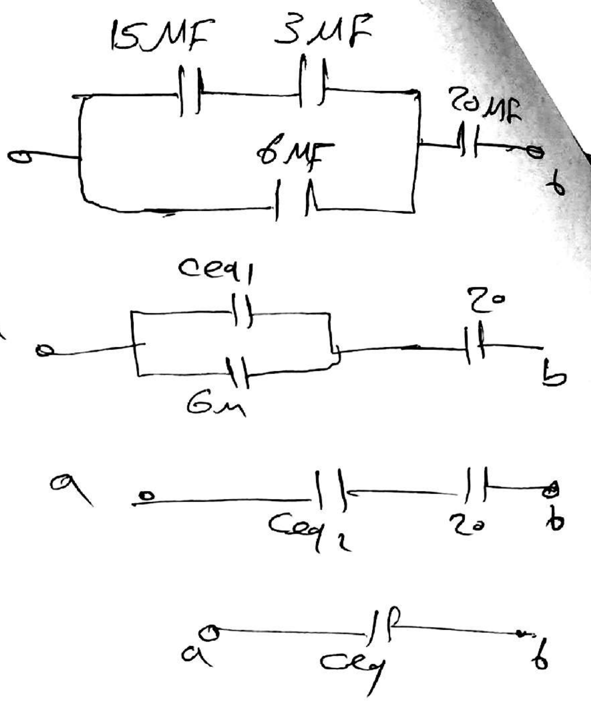
لو المكثفات متساوية identical

$$\boxed{C_{eq} = nC}$$

عكس كل المقاومات

EX(2)

a- Find equivalent capacitance bet^n a, b



$\frac{1}{Ceq_1} = \frac{1}{3} + \frac{1}{15} = \frac{5+1}{15}$

$ceq_1 = \frac{15}{6} = 2.5\mu f$

$ceq_2 = 6\mu + Ceq_1 = 8.5\mu f$

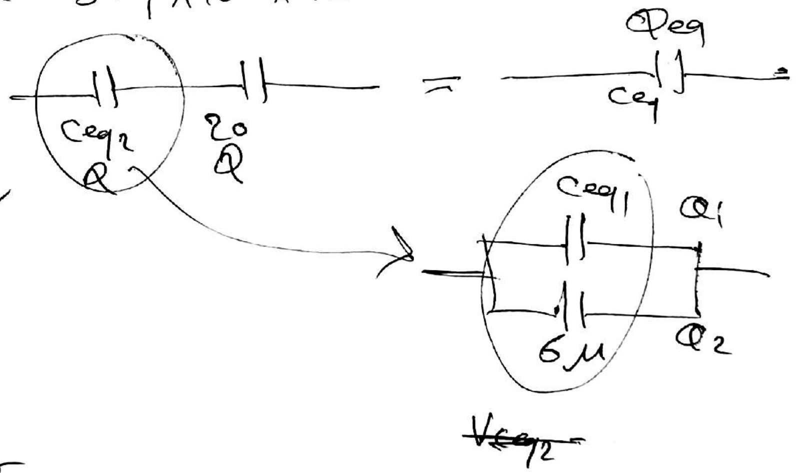
$\therefore Ceq = \frac{1}{8.5\mu} + \frac{1}{20\mu} \Rightarrow 5.9\mu f$

ترسم كل جزء جزء لا تتغير أشكال

b- مطلوب ثاني

if $V_{ab} = 15v$ find charge on each capacitor.

Sol/ $Ceq = \frac{Q}{Veq}$, $Q_{eq} = 5.9 \times 10^{-6} \times 15 = 88.5\mu C$



$\therefore Q_{20} = Q_{ceq_2} = 88.5\mu C$

$V_{ceq_2} = \frac{Q}{ceq_2} = \frac{88.5\mu}{8.5\mu} = 10.5v$

$V_{20} = \frac{88.5\mu}{20} = 4.5v$

$Q_1 = Ceq_1 * V_{ceq_1}$

$V_{ceq_1} = V_{6\mu} = Ceq_2 = 10.5$

$, Q_1 = \underset{ceq_1}{2.5\mu} \times 10.5 = 26.25\mu c$

$Q_2 = 6\mu \times 6.5 = 39\mu c$

Note $Q_1 = Q_{15\mu} = Q_{3\mu} = 26.25\mu f$

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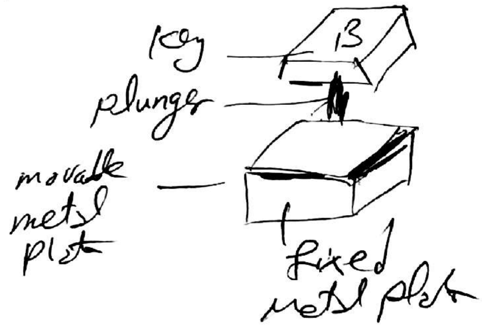
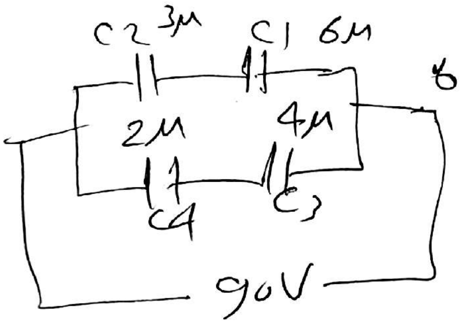
تطبيق عملي

The keyboard of computer represent set of capacitor

عند الضغط على الزر فيتم تغير المسافة بين اللوحين فيتغير
سعة المكثف فتتغير عدد الالكترونات كما يسبب
تغير حدث معين بالنسبة للجهاز



EX(3) If $V_{ab} = 90V$ find V_1, V_2, V_3, V_4



$C_{eq} = (6\mu \,//\, 3\mu) + (4\mu \,//\, 2\mu)$

$1/(\frac{1}{6\mu} + \frac{1}{3\mu}) + 1/(\frac{1}{4\mu} + \frac{1}{2\mu})$

$= 3.33\,\mu F$

$\cancel{V_{ab} = \frac{Q}{C_{eq}} \Rightarrow Q = C_{eq} V_{eq} = 3.33 \times 90 =}$

$\cancel{Q_{eq} = Q_{2\mu}}$

$\cancel{Q_{2\mu} = Q_{4\mu} = C_{2\mu} \times V_{2\mu} = C_{4\mu} \times V_{4\mu}}$

$\cancel{V_{2\mu} = \frac{Q_{2\mu}}{C_{2\mu}}}$

$V_1 + V_2 = V_3 + V_4 = 90 \rightarrow$ (1)

$Q_{6\mu} = Q_{3\mu} \therefore C_1 V_1 = C_2 V_2 \quad (6, 3)$ $\therefore \boxed{V_2 = 2V_1} \rightarrow$ (2)

$\therefore V_1 + V_2 = 90$ or $V_1 + 2V_1 = 90 \Rightarrow 3V_1 = 90$

$\Rightarrow V_1 = 30$

$V_2 = 60V$

$V_3 + V_4 = 90$, $C_3 V_3 = C_4 V_4$

$4V_3 = 2V_4$ $\boxed{V_4 = 2V_3} \rightarrow$ (3)

$\therefore V_3 + 2V_3 = 90$ $\Rightarrow V_3 = 30$

$V_4 = 60$